Continuous Random Variables
Day 1
Sec. 7.1 & (part of) 7.2

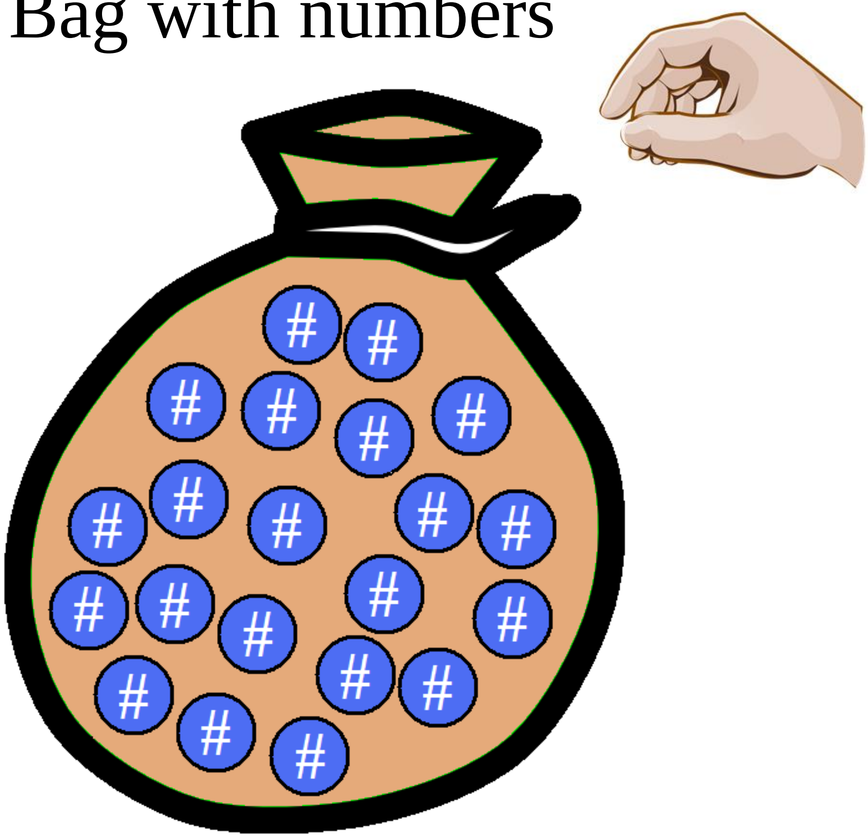
How Should You Think of a Random Variable?

Imagine a bag with numbers in it. You draw one number from the bag and the random variable X is the number that you drew from the bag.

- How did the bag with numbers get there? From some experiment in the background whose outcomes we relabeled with numbers. But the end product is the bag with numbers and that's what we care about.
- If you know the probability distribution of X then you can answer questions about what numbers can be drawn from the bag and what the chances are (probability) that various numbers are drawn from the bag.

How Should You Think of a Random Variable?

Bag with numbers

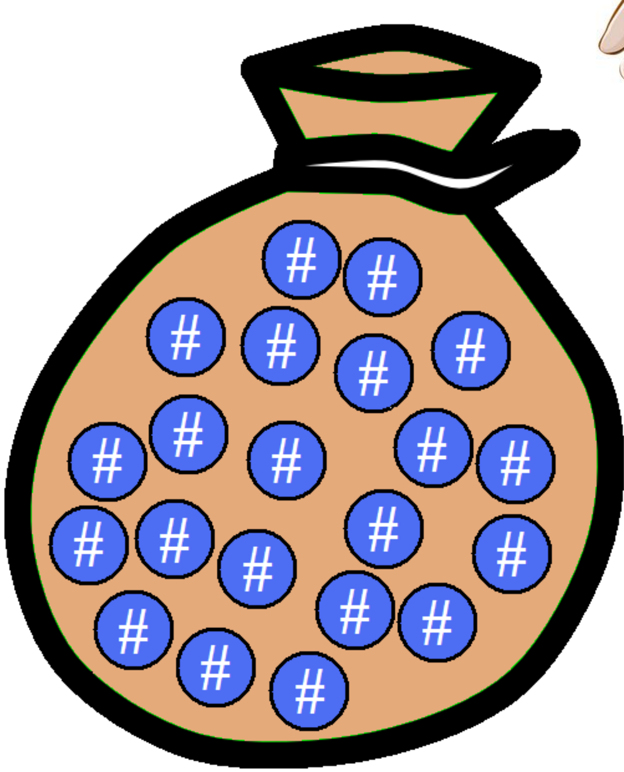


The experiment in the background could be…

- Betting on the draw of a card from a deck
- Counting the total number of heads when you flip a coin 4 times
- or something else

How Should You Think of a Random Variable?

Bag with numbers



If you know the probability distribution, say…

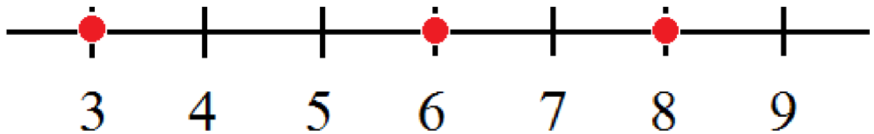
x	0	1	2	3	4
P(X=x)	0.05	0.15	0.10	0.50	0.20

Then I know

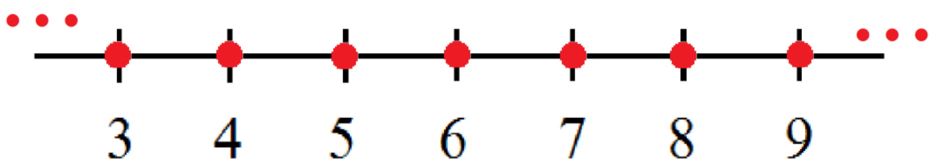
- The possible numbers that can be drawn from the bag
- The probabilities for the different numbers that can come out of the bag
- The average (EV), standard deviation and variance of all numbers in the bag

Continuous vs. Discrete Random Variables

Discrete Random Variable → The possible values of *X* are finite or countably infinite

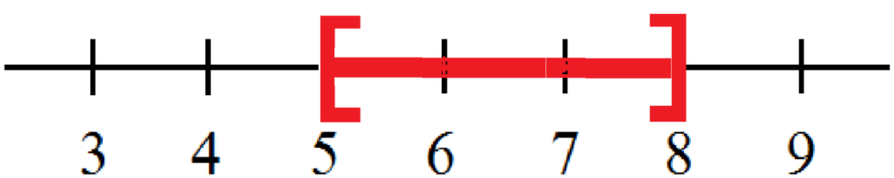


Possible values are 3, 6, & 8



Possible values are all integers

Continuous Random Variable → The possible values of *X* contain an interval



Possible values are ALL numbers between 5 and 8

Possible values are ALL REAL NUMBERS

Continuous Random Variables

For a continuous random variable…

- A probability distribution is a CURVE (called a density curve).
- Possible values of X are the numbers on the x-axis for which there is a curve above them.
- Probabilities are AREAS
    - To find a probability, find the area under the curve, above the x-axis between the 2 given values.

Requirements of a Probability Distribution for a Continuous Random Variable

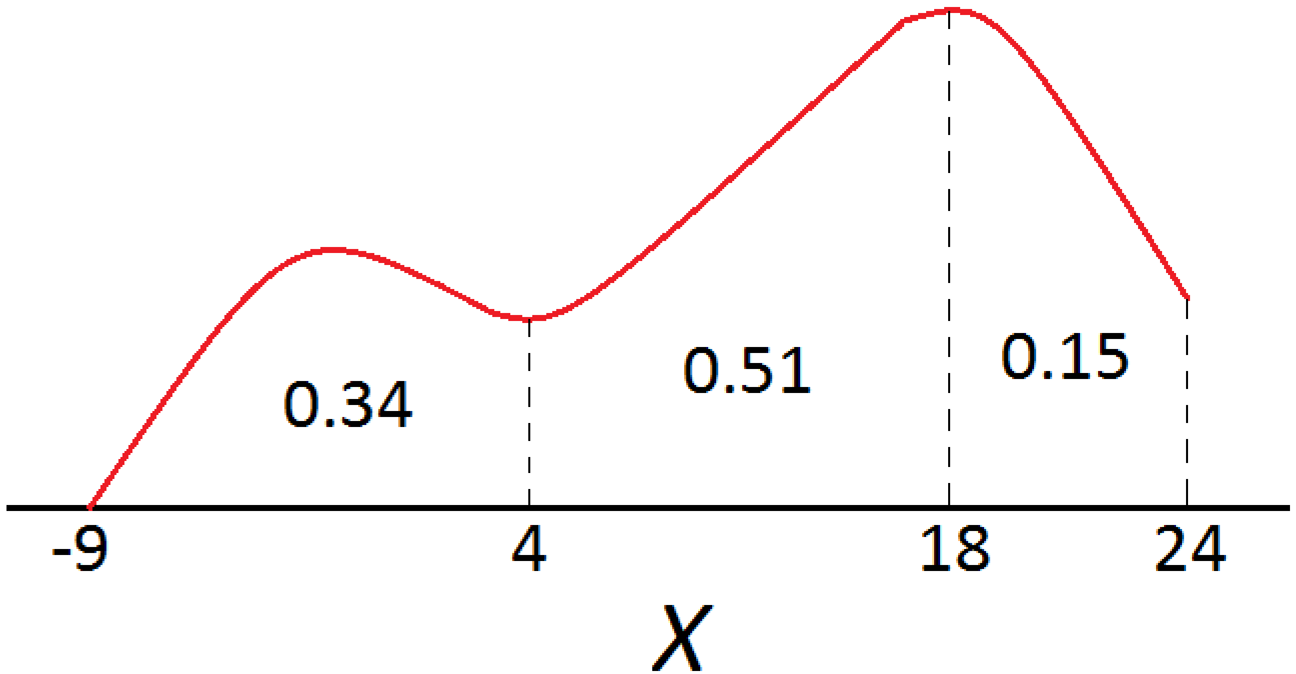
1. The curve must be above or on the x-axis

2. The total area under the curve must equal 1

Comparing Discrete & Continuous Random Variables

	Discrete	Continuous
Probability Distribution	Table	Curve
Calculating Probabilities	Go back to sample space	Area under curve
Prob. Dist. Requirement 1	$0 \leq P(X = x) \leq 1$	Curve on or Above *x*-axis
Prob. Dist. Requirement 2	$\sum P(X = x) = 1$	Total Area Under Curve = 1

Ex 1: Let X be a random variable with the following density curve.



a) What are the possible values of X?

b) Find $P(18 \le X \le 24)$

c) Find $P(4 \le X \le 24)$

d) Find $P(-13 \le X \le 18)$

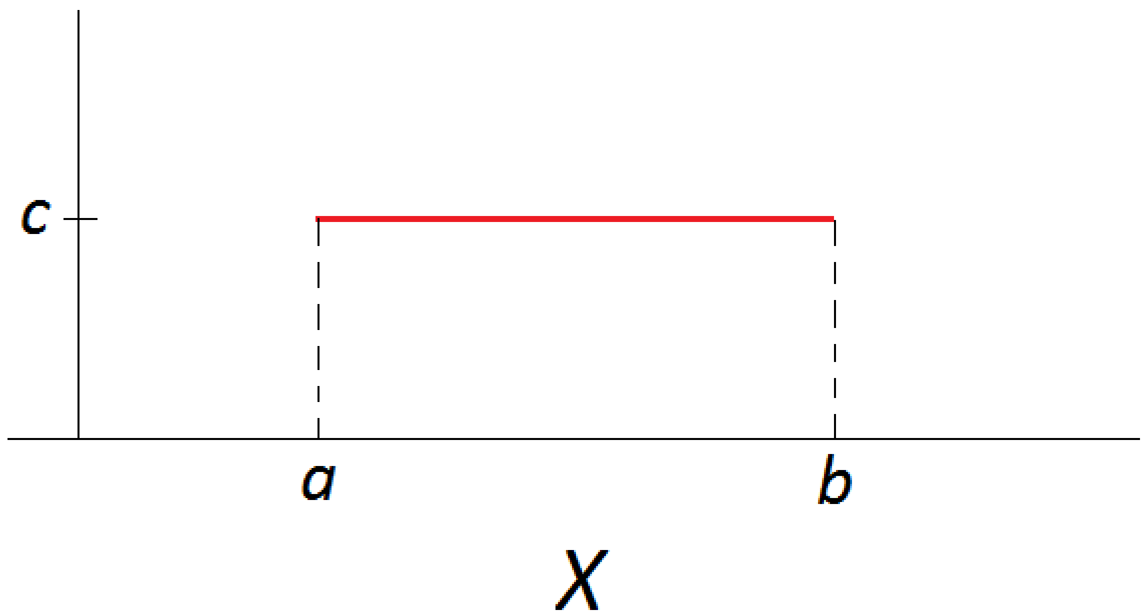
e) Find $P(X = 4)$

f) Explain the meaning of the probabilities found in (b)-(e)

The Uniform Distribution Over [a, b]

The things to remember for a uniform distribution are…

- The density curve is a straight horizontal line over the given interval [a, b]
- c will denote the height of the curve above the x-axis



Ex 2: Suppose X has a uniform distribution over the interval [3, 15].

a) Find the value of c that makes this curve a probability distribution

b) Find $P(3 \leq X \leq 7)$

c) Find $P(8 \leq X \leq 10)$

d) Find $P(10 \leq X \leq 22)$

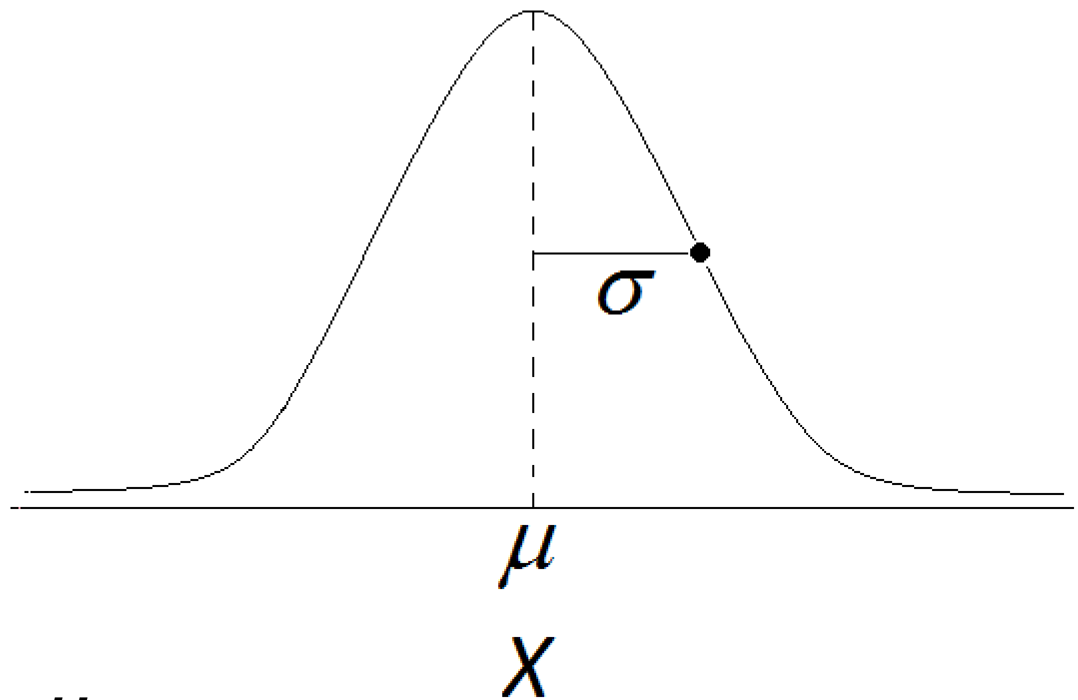
e) Find $P(X = 7)$

f) Explain the meaning of the probabilities found in (b)-(e)

Normal Distributions

The random variable *X* has a normal distribution if its density curve is the function…

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



The expected value of *X* is μ

The standard deviation of *X* is σ

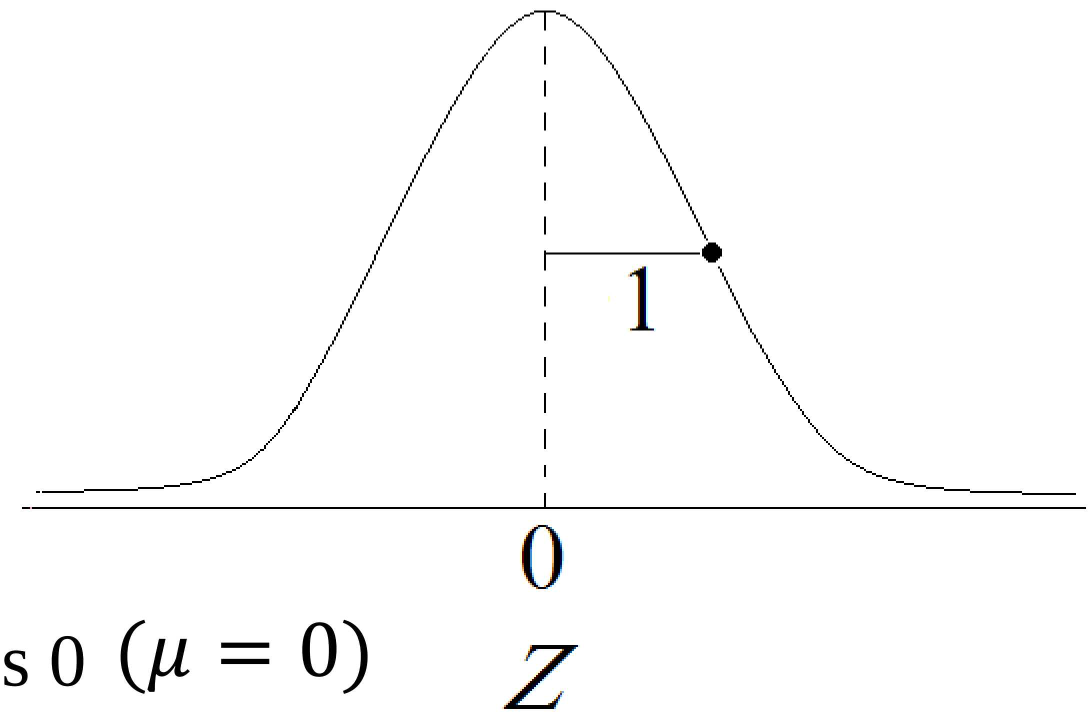
The possible values of *X* are all real numbers $(-\infty, \infty)$

There are many different normal distributions

The Standard Normal Distribution

The standard normal distribution is the normal distribution where the mean μ is 0 and the standard deviation σ is 1.

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$



The expected value of Z is 0 $(\mu = 0)$

The standard deviation of Z is 1 $(\sigma = 1)$

The possible values of Z are all real numbers $(-\infty, \infty)$

For the random variable with a standard normal distribution, we'll use Z instead of X